

# exam-n: exam papers

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This class file supports creating exam papers. This is intended as part of a slightly larger system for assembling and managing the papers, by combining questions from multiple authors.

This document is addressed to the exams convener, or someone responsible for the overall exam; it goes into complete detail about the class file, and how to customise it. There is more compact documentation for the class, addressed to the authors of individual questions, in the companion document 'Notes for Authors'.

The most up-to-date version of this class can be found at <http://purl.org/nxg/dist/exam-n>.

# 1 Description

Usage:

```
\documentclass[options...]{exam-n}
<preamble declarations>
\begin{document}
\maketitle
\begin{question}...\end{question}
...
\end{document}
```

## 1.1 Class options

The class takes the following options. Note that these options are processed in order, so if an option changes a default (for example, **draft** sets **showsolutions** by default), then it can be overridden by an option later in the list.

**compose, draft, final** When composing questions, you should give the **compose** option to `{exam-n}`; when assembling the paper, you should use **draft**; and you should use **final** only for the final version. The **draft** option switches on the **showsolutions** option (below), displays `\comment` remarks, and makes some mild layout changes. The **compose** option implies the **showsolutions** option, causes questions to be formatted one per page, and turns off various checks on the number of questions; the **draft** option should have the same pagination as the final paper and shifts the body of the paper to the left so that marginal comments are easier to write and read; the **final** option turns off the `\comment` command.

**cmfonts, psfonts, mathptm, mathtime, mtpro2** The **cmfonts** option uses the Computer Modern fonts for the document, and the **psfonts** uses PostScript fonts. In this latter case, you will possibly want to use one of the **mathtime** or **mtpro2** options also, to use the **mathtime** or **MTPro2** font set for the maths in the document; alternatively you can use the **mathptm** option to use that free but less good alternative.

**(no)showsolutions** These control whether the solutions to the questions are shown on the paper or not. **showsolutions** is the default when the either **draft** or **compose** options is present, and **noshowsolutions** in **final** case, though this will be overridden by one of the `{no}showsolutions` options later in the list.

**(no)perquestionmarks and (no)showmarktotals** These control whether mark totals and subtotals are tracked within the question (**perquestionmarks**), and whether the total available marks are displayed at the end of the question (**showmarktotals**). See Sect. 1.2 for more discussion (this option was called **showmarks** up to release 0.15).

**(no)pageperquestion** If the option `pageperquestion` is present, then each question is on a separate page. Option `nopageperquestion` is the default. This is most often set by default by one of the other options, or by a `.c1o` file.

**oneside, twoside** These control whether the document is formatted for one- or twosided printing. This is a standard option, which is redundant in this case, since in this document style there is no difference in formatting. You may in principle use other article options, though you are encouraged not to.

**fleqn** Display equations flush-left, rather than centred. You generally won't set this in the `\documentclass` options – it's here so that it can be set in a `.c1o` file.

**sloppydescription** Disables the requirement that all exam metadata be present. See the discussion of `\universitycoursecode` below.

**mono, colour** The university logo is typeset in colour by default, as is the solution text; the `mono` option causes everything to be in mono instead. The `colour` option does nothing, but is present for symmetry.

**sansserif** Use a sans serif font for the body text. This is plug-ugly, not least because it doesn't match the maths font; also, it may be easier or harder for dyslexic students to read (depending on your superstition – there seems remarkably little actual evidence either way).

**bigfont** Produce a version of the paper in a 'huge' font (36pt size) for the benefit of students with visual impairments. See notes below.

Any other options are interpreted as an instruction to read in a `.c1o` file, containing course-, department- or group-specific style modifications. Most typically, these changes will affect the rubric, and the sheet of physical constants. The only generic style option is `A1` (which is included as an example of how such a file is written). See section 2 below.

In some circumstances – for example when processing exam scripts under the control of a Makefile or script – it can be convenient to control package options from outside the package. If there is a file called `exam-n.config` in the input path (most likely in the same directory as the exam paper), then this is read in when any exam script is processed, and will supplement, any options in the `\documentclass` line. For example, if this file were present and contained the line `\ExecuteOptions{showsolutions}`, then the typeset exam would include the solutions. Note that this will supplement, *but not override*, options in the input file's `\documentclass`; it is therefore useful only for switching options away from the default.

The `exam-n` class includes the `{amsmath}` package, so you can make immediate use of `amsmath` features if you wish (see <http://www.ams.org/tex/amslatex.html> for discussion).

The change of font size with the `bigfont` option requires a couple of minor layout changes. You may need to make some other changes by hand, in this

case. You can do that by bracketing the adjusted text in `\ifbigfont<big font material>\else<normal font material>\fi`. The `{amsmath}` `multline` environment can be useful here. The exams convener should review the result carefully: a few judicious `\ifbigfont\newpage\fi` insertions can make the result look less awful. The `bigfont` option applies to the content of solutions if the `showsolutions` option is present; this may or may not be the optimal choice.

At the bottom of each page, you see a faint identification code, such as ‘QM/123-456’. This consists of an exam identifier, extracted from the exam preamble, plus a code which changes each time  $\LaTeX$  is run. This helps you avoid collation accidents, and to distinguish between slightly different versions of the printed document. The identifier is based on the date and time, so changes each time you run  $\LaTeX$  on the file (that is, it identifies a printing, rather than a source-file version).

## 1.2 The Question environment

Within the document, you include questions within a `{question}` environment, within which you may further have `{questiondata}` and `{solution}` environments.

`question`      The `{question}` environment delimits a single exam question.  
Usage, with `perquestionmarks`:

```
\begin{question}[\langle questionnumber \rangle]{\langle marks \rangle}
...
\end{question}
```

Or, with `noperquestionmarks`:

```
\begin{question}[\langle questionnumber \rangle]
...
\end{question}
```

There are two variants of this environment, depending on whether the `perquestionmarks` option is present or not. If it is present (the default) then the `{question}` environment takes a non-empty argument showing the total marks available for a question. In addition, the class checks that the `\partmarks` within the question add up to this declared goal mark.

If the option is not present, then questions have no individual marks, the environment takes no marks argument, and no marks are shown (this is usual in essay-question exams, which typically comprise a sequence of short equal-valued questions with an explanatory rubric). The `\partmarks` command cannot be used within the question. If the `showmarktotals` option is present, then this (expected and checked) total is displayed at the end of the question.

The `{question}` environment takes an optional argument, giving the expected question number. In `compose` mode, this is used as the question number (unsurprisingly). In the two other modes, this is compared with the expected question number, based on the number of preceding `{question}` environments in the file,

so that the first `{question}` environment is for question one, the second for question two, and so on. If these do not match, the exam class displays a warning in `draft` mode, and produces a fatal error in `final` mode. You should generally give this argument – it is intended to help catch embarrassing errors.

In some cases, the questions are not numbered in this straightforward fashion, so that you might have questions ‘2A’ and ‘2B’. In this case, the check is not meaningful, and you can suppress it by calling the macro `\QuestionNumberChecksOff` in the preamble, and the question-number argument is required (though it must still appear within square brackets). If these checks are off, but a `{question}` appears without a forced question number, then the class warns about this, but still numbers the question as the next one in sequence.

**solution** The `{solution}` environment, contained within the `{question}` environment, contains the solution to the question, or other notes. It is displayed by default in `compose` mode, and suppressed by default in the others, though this behaviour may be overridden in either case with the `(no)showsolutions` option. You can have one `{solution}` at the end of your question, or have multiple ones scattered throughout it. You may use the `\partmarks` macro within the solution, to indicate the distribution of marks within (this part of) the solution – these, of course, do not count towards the total mark for the question. You must not have a solution inside a solution.

**questiondata** At the end of a question, it is frequently useful to include further information, such as extra equations, or numerical data. These should be included within a `{questiondata}` environment, in order for them to be formatted appropriately. You may include multiple paragraphs, equations, and displays in this environment, as appropriate. Typically, you will have only one such environment per question, appearing at the end, but may have several of them if you really wish to.

**mcq** Some exams include multiple-choice questions rather than extended-answer ones. These are numbered in the same sequence as the other questions, but are formatted and marked-up differently.

```
\begin{mcq}
In 1908, where was there an airburst ‘impact’?
\answer Tunguska
\item Arizona
\item Off the Mexican coast
\item Egypt
\end{mcq}
```

That is, the `{mcq}` environment contains a list of possible answers, all of which are indicated by `\item`, except precisely one correct answer, indicated by `\answer`. All multiple-choice questions must have the same number of possible answers, which is declared with the command `\multiplechoiceanswers{n}`. The `{mcq}` environment is permitted only after `\multiplechoiceanswers`. It’s OK to have a `{solution}` within an `{mcq}` environment, which might provide further commentary on the correct answer.

**\multiplechoiceanswers**

There are various other commands which you may or should use within the document.

`\includequestion` It may be convenient to split your exam into a number of separate source files, such as having one `.tex` file for each question. You can include these various source files using the usual `\input` command.

If the separate source files have the simple form:

```
\documentclass[compose]{exam-n}
\usepackage{graphicx} % for example
\begin{document}
\begin{question}
...
\end{question}
\end{document}
```

then they can be L<sup>A</sup>T<sub>E</sub>Xed separately, for example by the authors of different questions, but cannot be `\input` unedited, as described above. If, however, they have *only* these structures (that is, only the `\documentclass` command, the `{document}` environment, and zero or more `\usepackage` commands), then you can most conveniently import them unedited using the `\includequestion` command.

```
\includequestion{dynamics2}
```

This acts like the `\input` command, but disables the listed structures. It also puts the included command into a group, so that any (re)definitions of commands are made local-only.

Recall that the definition of the `\includequestion` command means that any `\usepackage` commands will be ignored. If you, as a question author, need certain packages to be present for your question, you will have to make sure that whoever is assembling the master file includes those packages there, too.

The `\includequestion` command takes an optional argument which overrides the question number. This caters for the case where question authors have (unhelpfully) included question numbers in the files' `{question}` environments, and the case where questions are not numbered in a straightforward sequence, for example '1', '2A', '2B', and so on.

`\section`  
`\subsection` Some exams are divided into sections, or have other structure which needs to be spelled out. These are described with the `\section` command, in a form such as `\section{II}`. You can also add smaller headers before individual questions with something like `\subsection{Second semester questions}`. These can appear only between questions; it is an error to include one of these commands within a `{question}` or `{mcq}` environment.

`\part` Questions may be subdivided into parts, such as (a), (b), (c)..., or (i), (ii), (iii), and so on. Precede each of these with this `\part` command. The formatting of the part numbers is controlled by the exam style, as customised in Sect. 2. This macro starts a new paragraph.

`\partmarks` Macros `\partmarks{<num>}` and `\partmarks*{<num>}` announce the number of marks associated with the current part of a question. The class checks that the

number of marks here does add up to the number declared at the beginning of the question environment. You will typically have just one `\partmarks` per `\part`, but you can have more if you want. The starred version should be used when a part ends with an unnumbered equation, as it adjusts the spacing appropriately; it might also be useful at the end of a part which finishes with an itemized list, or some other structure which leaves an expanse of white space on the right-hand side of the page. The unstarred version should only be used at the end of a paragraph, and in fact forces a paragraph end; the starred version should generally be used only at the end of a paragraph, but it doesn't force one.

`\comment` `\comment{<text>}` associates a comment with a part of the text. This is ignored in `final` mode, but appears in the margin in the other modes.

`\author` The `\author` command is a convenience. Used within a `{question}` environment – most naturally just after the `\begin{question}` – it creates a comment with the author's name. Its functionality may be expanded in future, so you should use this command, rather than a generic `\comment`, when noting the authorship of a question.

`\shout` If there is part of a question which is, for example, incomplete, and which needs a more prominent callout than 'comment', then you should `\shout{...}` it. Shouts appear in all modes (including `final`) and appear whether or not the class is showing solutions. This makes a prominent remark in the text, and also in a list of shouts at the end of the text. Your co-authors really have no excuse for missing it after that.

`\leftnudge` For various reasons, most often because of printing problems, it can be useful to nudge the textblock left or right a little. You should call the `\leftnudge` command to do this, rather than fiddling with the underlying L<sup>A</sup>T<sub>E</sub>X dimensions yourself. Give the command a dimension argument such as `\leftnudge{1cm}` to nudge the textblock leftwards by 1cm. You can give a negative dimension to nudge it rightwards instead. The `draft` option automatically nudges the text block leftwards, to create a larger right-hand margin for notes.

`\questionpreamble` If `\questionpreamble` is called, then its contents are displayed just before the start of the next question. This is useful for text like `\questionpreamble{And one of\dots}` which might reinforce information in the examination rubric.

### 1.3 Preamble

`\exambanner` The `\exambanner` macro supplies text like 'Examination for the degrees of...'. Since the contents of this command is automatically uppercased in some styles, and there are per-department specifics about the punctuation of abbreviations, you should use the commands `\BSc`, `\MSci`, and friends (see section 1.4) to set the degree names appropriately.

`\universitycoursecode` Declare the identity of the exam with `\universitycoursecode`, `\schoolcoursecode`,  
`\schoolcoursecode` `\coursetitle` and `\degreedescriptions`. The distinction between these is as  
`\degreedescriptions` follows:  
`\coursetitle`

**University course code** This is the code for the course (and thus for the paper) as it appears in university information systems (for example, MyCampus),

and is a university-unique code such as ‘PHYS3031’.

**School course code** This is a more informal, but more recognisable, code for the course/paper, as it is generally recognised within the school; for example, the honours Quantum Mechanics course is known within the school as P304H.

**Course title** This is just a textual name for the course, for example ‘Quantum Mechanics’.

**Degree descriptions** This is a textual description of the qualifications that the students doing this exam are heading for. This text has little formal weight, but might help a lost student realise they’re in completely the wrong exam room... This is something like `{Physics 3\Chemical Physics 3}` (separate each description using `\\`).

All of these are required elements, and the  $\text{\LaTeX}$  compilation will halt if they are absent. If for some reason the exam paper does not need these to be present – perhaps it is a class test, for example – then give the class option `sloppydescription`, and the checks for these elements (and for `\rubric`) are suppressed. The layout may end up looking a little funny.

`\paperident` It can be convenient to add some identification to each page, if for no other reason than to double-check that you haven’t inserted a field theory question into an ExCos exam. The command `\paperident` allows you to declare some text which appears at the bottom of each page of the exam. It will typically repeat some of the text in the `\schoolcoursecode` or `\degreedescriptions` arguments. This is generally not necessary, however, as in its absence the class generates an identifier. This identifier contains the name of the exam, plus a pair of counters (for example *QM2/98-1177*). The function is two-fold: (i) since the counters increase monotonically (they actually encode the date and time when the document was  $\text{\LaTeX}$ ed), you can tell which of two superficially similar documents is the later; and (ii) if you drop a sheaf of papers on the photocopier floor, you can work out which one is which.

`\examdate` Give the date and time of the exam with `\examdate` and `\examtime`. Sometimes an exam may have different time limits for different qualifications: this case, separate the various times with `\\`, as in `\examtime{9.30am -- 12 noon\\(or) 9.30am -- 1.45am}`

`\rubric` The rubric, provided unsurprisingly by the command `\rubric`, may contain more than one paragraph, delimited by the usual blank line. Any emphasised words should be marked with `\emph` – they are typically emphasised with a bold font. The class checks that a rubric has been specified (unless `sloppydescription` is present); if you really wish to suppress this rubric – perhaps because

`\baserubric` the `\baserubric` is sufficient – then give the command `\norubric`. As well as this exam-specific rubric, the style produces an additional boilerplate rubric, containing the usual material such as ‘Do not on any account attempt to write on both sides of the paper at once. Calculations may be done on the fingers, but candidates should avoid counting on their toes unless special permission has been obtained beforehand.’ You will typically not have to change this, but if you do for some reason, you can override it with the `\baserubric` command.



---

<code>\e</code>	$e^{i\pi} = -1$	the exponential is typeset in an upright rather than italic shape, as in <code>\e^{i\pi}=-1</code> .
<code>\au</code>	10au	use this macro for astronomical units.
<code>\lambdabar</code>	$\lambda$	the Compton wavelength, as a lambda with a bar through it.

---

Table 1: Miscellaneous symbols

---

`\numquestions` Finally, declare the number of questions which are to be in the paper with `\numquestions`. The class issues a warning if we don't have this number, in draft or final mode. This is optional – no check is done if this isn't present.

## 1.4 Other useful commands

`\BSc and friends` Macros `\BSc`, `\MSci`, `\MSc`, `\MA`, `\MEng` and `\BEng` are used within the preamble macros to give appropriately capitalised and punctuated versions of the degree types.

`\vec` Macro `\vec` is redefined to give bold-font vectors, rather than vectors with arrows, which is the (weird) L<sup>A</sup>T<sub>E</sub>X default. This should work for bold greek as well as roman.

`\dd` Macros `\dd` and `\ddd`: `\dd` is a roman d, as used for differentials; `\ddd` is the same with a preceding thinspace, as used within integrals; for example

$$\int f(x)\ddd x = \int f(x)\, \dd x = \int f(x) dx$$

`\Diff1` You can typeset derivatives neatly:

<code>\Diff1{a}{b}</code>	$\frac{da}{db}$
<code>\Diff1[2]{a}{b}</code>	$\frac{d^2a}{db^2}$
<code>\Diff1*{a}{b}</code>	$da/db$
<code>\Diff1*[2]{a}{b}</code>	$d^2a/db^2$

The unstarred versions are for displayed equations, the starred ones for inline maths. There is analogous support for partial derivatives with `\Partial`.

`\units` Macros `\units`, `\units*`: you can typeset physical units in `\rm`, with tilde or dot acting as a separator between units. Since this is typeset in maths mode, all other spacing is ignored. The unstarred version includes leading `\thinspace`, as in `$v=10\units{m.s^{-1}}$`, giving  $v = 10 \text{ ms}^{-1}$ . The starred version can be used when referring to the unit by itself (eg axis is `$B/\units*T$`, or  $B/T$ ), and is not qualifying a number. These macros might be replaced, in the future, by a recommendation to use the `{siunitx}` package.

For other useful symbols, see table 1.

## 2 Customising the exam style

As described in section 1.1 above, any unrecognised options are interpreted as an instruction to search for and include a class options file, formed from the name of the unrecognised option, suffixed with `.clo`, which can be anywhere in the  $\TeX$  include path. This options file has a fair amount of leeway to override and adjust the layout of the exam.

The most typical changes here will be to adjust the exam rubric for a particular class, with the command `\baserubric`, and to change the sheet of physical constants, with the command `\constantssheet`. See the file `A1.clo` for examples.

Examine this sample `SpecialExam.clo` file:

```
\typeout{Physics Special exam options, for Special people}
\ExecuteOptions{pageperquestion}

\OverrideFormatting{
\renewcommand\FormatPartMarks[1]{\{#1\}}
\renewcommand\FormatPartNumber
  {\hbox to 0pt{\hss (\alph{partnumber})\hskip1em}}
\renewcommand\FormatQuestionNumber
  {\hbox to 0pt{\hss \textbf{\@currentquestion}\hskip2.5em}}
}

\constantssheet{
  \begin{center}
    $E=mc^2$ and $c=3\times 10^8 \text{ \rm m}\,s^{-1}$
  \end{center}
}
```

This announces itself, then invokes the exam style's `pageperquestion` option.

It then includes a number of formatting adjustments, enclosed within the `\OverrideFormatting` command; the formatting hooks are described below.

Then it declares a 'constants sheet', which is a display of constants or equations, or indeed anything else, which is to be displayed on the second page of the exam.

The available formatting hooks are as follows:

**FormatPartMarks** This formats the indication of the marks carried by a particular part of a question. In this case, we have chosen to have the marks contains inside curly brackets, rather than the default square brackets. By default, the part-marks text will be placed at the end of the paragraph it completes, flush right, and with at least 2em of space before it. You can change this default space with `\@partmarksspace`; as a special case, you can have the text sitting in the margin instead, by having `\FormatPartMarks` generate a zero-width box, and setting `\@partmarksspace=0pt`.

**FormatPartNumber** This overrides how to format the various `\part` markers within a question, using the `partnumber` counter. In this case, the markers will jut into the left-hand margin, rather than being run-in.

**FormatQuestionNumber** If you adjust the part marks, you should probably adjust the formatting of the question number also.

If you really want to go to town on reformatting, you can redefine the command `\maketitle`, which formats the front-page title. When formatting this, you have access to each of the fragments of text described in section 1.3, via a macro named after the corresponding command. Thus the argument of the `\exambanner` command is available in the macro `\@exambanner`. The exception is `\numquestions`. If you find yourself needing to do this, it might be worth having a discussion with the style's maintainer – there may be a simpler way to get what you want.

No more clues. If you want to hack at this, see the definition of `\maketitle` in the class file `exam-n.cls`. Aspire not to break things.

### 3 A work-flow

This class file was developed as part of a move to ‘L<sup>A</sup>T<sub>E</sub>Xify’ the production of exams in the School of Physics and Astronomy, in the University of Glasgow. The work-flow which emerged was, roughly, as follows:

1. The exam convener (the person in charge of assembling the exam for a given course or lecture series) creates a Sharepoint directory which contains a master file (which consists mostly of `\includequestion` macros), plus subdirectories each containing a short template file for the benefit of question authors, and a copy of the `notes-for-authors.pdf` instructions.
2. Each of those ‘question’ directories is made visible to, and read-writable by, the lecturer responsible for that question. They write and L<sup>A</sup>T<sub>E</sub>Xed their question, and upload it back to Sharepoint.
3. The exam convener, who has access to the whole tree of questions, downloads or network-mounts the Sharepoint folder, and either edits the questions or negotiates with the authors, until the exam L<sup>A</sup>T<sub>E</sub>Xs successfully. The result can then be snapshotted, and sent off to the external examiner.

This provides a centralised and easily manageable repository for a previously fiddly and error-prone process.

In step one, the template file is just one with a `[compose]` option, following the brief example on p.6. We had excellent secretarial help to manage this step; it was fiddly the first couple of times, but largely mechanical thereafter. In Sharepoint, this step can in principle be automated, though we concluded this was unnecessary in our case.

Sharepoint (possibly slightly surprisingly) worked smoothly for us; in other circumstances a group-writable source code repository, or an institutional CMS, or even just a shared drive, would work just as well. The advantage of Sharepoint, from our point of view, was that it integrated with the university's existing authentication framework, and was (axiomatically) acceptably secure for the process of creating exams.

The majority of our authors were habitual users of L<sup>A</sup>T<sub>E</sub>X for writing journal articles, and with subsequently submitting them to journals or the arXiv; and when the process was presented in those terms, with the same advantages, it was readily accepted.

Thanks to Graham Woan and Rachael MacLaughlan for the numerous iterations of this process until both it and the class file were stable.

## 4 Example

Here is a short example file. There are further examples in the `sample/` directory of the distribution.

```

1 \example
2 %%START example (Makefile strips out this block)
3 \documentclass{exam-n} % standard final version
4 %\documentclass[draft,showsolutions]{exam-n} % draft style, showing solutions
5 %\documentclass[compose]{exam-n} % compose (author's) style
6
7 \examdate{Wednesday, 18 May 2005}
8 \examtime{9.30am -- 12 noon\\(or) 9.30am -- 1.45am}
9
10 \exambanner{Examination for the Degrees of \BSc(Science) and
11 \MSci\ on the Honours Standard}
12 \schoolcoursecode{P304D and P304H}
13 \universitycoursecode{PHYS3031 and PHYS4025}
14 \coursetitle{Quantum Mechanics}
15 %\degreedescriptions{Physics 3, Chemical Physics 3, Physics with
16 % Astrophysics 3, Theoretical Physics 3M, Joint Physics 3}
17 \degreedescriptions{Physics 3\\Chemical Physics 3\\Physics with
18 Astrophysics 3\\Theoretical Physics 3M\\Joint Physics 3}
19 \paperident{GR/P304}
20
21 \rubric{Candidates for examination in \emph{Quantum Mechanics} should
22 answer question 1 (16 marks) and \emph{either 2A or 2B} (24 marks each)}
23
24 \numquestions{3}
25
26 \begin{document}
27 \maketitle
28
29 \section{I}
30
31 \begin{question}{20}
32 \part At various points in the development of the mathematical theory of
33 General Relativity, we pick a coordinate system in which
34 differentiation is simple, and do a calculation using non-covariant
35 differentiation, indicated by a comma. We then immediately deduce the
36 covariant result, replacing this comma with a semicolon.
37

```

38 Separately, the strong equivalence principle is sometimes  
39 referred to as the ‘comma goes to semicolon’ rule.  
40

41 Explain the logic of each of these replacements of a comma with a  
42 semicolon, putting particular stress on the distinction between  
43 them.\partmarks{10}

44

45 \part The radial and angular coordinates,  $r$  and  $\phi$  respectively,  
46 of a test particle moving in the Schwarzschild metric exterior to a  
47 star of mass  $M$  and radius  $r$ , are related by the equation  
48 \[  
49 
$$r = \frac{h^2 M}{1 + e \cos \phi + \frac{3M^2}{h^2} e \phi \sin \phi}$$
  
50 
$$\left. \right)^{-1},$$
  
51 \]

52 \]

53 where  $h$  and  $e$  are constants. Show that this equation takes the  
54 form of a precessing ellipse, of semi-latus rectum  $l = h^2/M$ , in which  
55 the pericentre line advances each orbit by an amount  
56  $\Delta = 6\pi M^2/h^2$ , stating clearly any assumptions that  
57 you make.\partmarks{6}

58

59 The solar-mass star HD83443 has a 0.35 Jupiter-mass planet that  
60 follows a circular orbit of period 2.986 days and radius 0.038 au.  
61 Calculate the rate of precession, in arcseconds per year, of the  
62 pericentre line of the planet’s orbit.\partmarks{4}

63

64 [Schwarzschild radius of the Sun  $r_s = 3.0 \times 10^3$  m],  
65  $1 \text{ au} = 1.5 \times 10^{11}$  m].

66

67 \begin{solution}

68 In the first type of calculation, we do a calculation in the LIF, in  
69 which  $\Gamma^i_{jk} = 0$ , so that single partial differentiation is the  
70 same as covariant differentiation. If this process produces a  
71 geometrical object such as a scalar or a tensor, then we know that the  
72 result is frame-invariant. If the result involves only single partial  
73 differentiation -- that is, no second derivatives -- then since  
74 partial differentiation is the same as covariant differentiation in  
75 these coordinates, we cannot distinguish partial and covariant  
76 derivatives, and can replace the commas by semicolons. Since these  
77 are now manifestly covariant derivatives, so that the result is a  
78 tensor, and thus frame-invariant, the same expression would be true in  
79 any frame.

80

81 The second situation is the statement that the expressions of physical  
82 laws in SR, such as the conservation equation  
83  $T^{\mu\nu}_{;\nu} = 0$ , must take the same form when written  
84 as a covariant equation in GR, crucially without any curvature  
85 coupling. The slogan ‘comma goes to semicolon’ is just a mnemonic for  
86 this.

87

```
88 The distinction is that the first is a mathematical trick, of sorts,  
89 whereas the second is a version of the equivalence principle, and thus  
90 a statement with deep physical content.  
91  
92 They don't have to explain things at this length or with this  
93 coherence (?) to get quite a few marks. They just have to show they  
94 have a clue.  
95 \end{solution}  
96 \end{question}  
97  
98 \end{document}  
99 %%END example  
100 </example>
```

## Acknowledgements

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