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The Free High School Science Texts: Textbooks for High School Students Studying the Sciences Physics Grades 10 - 12

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Contents

Chapter 9

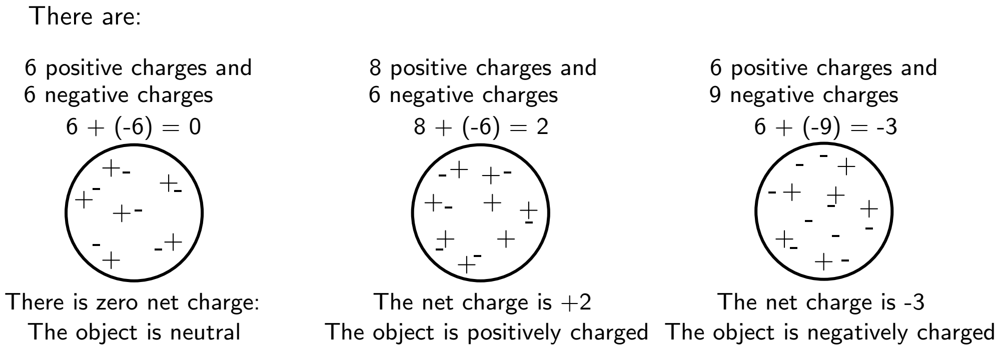
Electrostatics - Grade 10

9.1 Introduction

Electrostatics is the study of electric charge which is static (not moving).

9.2 Two kinds of charge

All objects surrounding us (including people!) contain large amounts of electric charge. There are two types of electric charge: **positive** charge and **negative** charge. If the same amounts of negative and positive charge are brought together, they neutralise each other and there is no net charge. **Neutral** objects are objects which contain positive and negative charges, but in equal numbers. However, if there is a little bit more of one type of charge than the other on the object then the object is said to be **electrically charged**. The picture below shows what the distribution of charges might look like for a neutral, positively charged and negatively charged object.



9.3 Unit of charge

Charge is measured in units called **coulombs (C)**. A coulomb of charge is a very large charge. In electrostatics we therefore often work with charge in microcoulombs (1 μC $= 1 \times 10^{-6}$ C) and nanocoulombs (1 nC $= 1 \times 10^{-9}$ C).

9.4 Conservation of charge

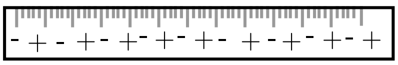
Objects can become charged by contact or by rubbing them. This means that they can gain extra negative or positive charge. Charging happens when you, for example, rub your feet against the carpet. When you then touch something metallic or another person, you will feel a shock as the excess charge that you have collected is *discharged*.

Important: Charge, just like energy, cannot be created or destroyed. We say that charge is **conserved**.

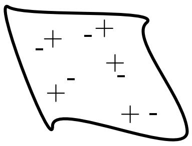
When you rub your feet against the carpet, negative charge is transferred to you from the carpet. The carpet will then become positively charged by the *same amount*.

Another example is to take two *neutral* objects such as a plastic ruler and a cotton cloth (handkerchief). To begin, the two objects are neutral (i.e. have the same amounts of positive and negative charge.)

BEFORE rubbing:



The ruler has 9 postive charges and 9 negative charges

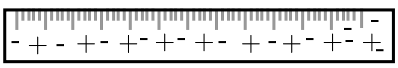


The neutral cotton cloth has 5 positive charges and 5 negative charges

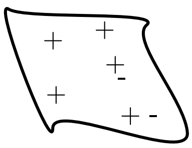
The total number of charges is:
(9+5)=14 positive charges
(9+5)=14 negative charges

Now, if the cotton cloth is used to rub the ruler, negative charge is transferred *from* the cloth *to* the ruler. The ruler is now *negatively* charged and the cloth is *positively* charged. If you count up all the positive and negative charges at the beginning and the end, there are still the same amount. i.e. total charge has been *conserved*!

AFTER rubbing:



The ruler has 9 postive charges and 12 negative charges
It is now negatively charged.



The cotton cloth has 5 positive charges and 2 negative charges.
It is now positively charged.

The total number of charges is:
(9+5)=14 positive charges
(12+2)=14 negative charges

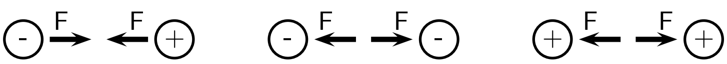
Charges have been transferred from the cloth to the ruler BUT total charge has been conserved!

9.5 Force between Charges

The force exerted by non-moving (static) charges on each other is called the **electrostatic force.** The electrostatic force between:

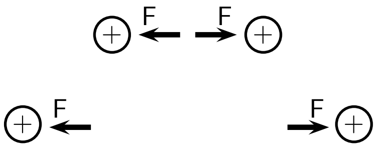
- **like** charges is **repulsive**
- **opposite** (unlike) charges is **attractive**.

In other words, like charges repel each other while opposite charges attract each other. This is different to the gravitational force which is only attractive.



attractive force repulsive force repulsive force

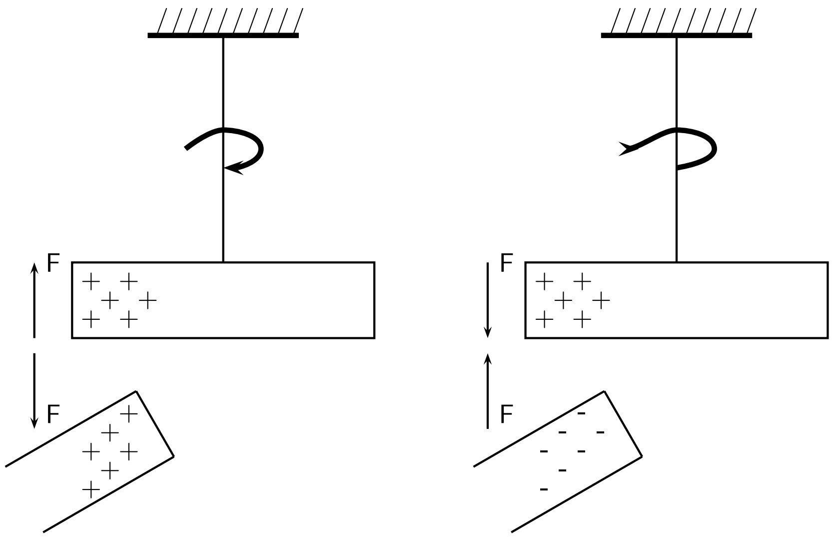
The *closer* together the charges are, the *stronger* the electrostatic force between them.



Activity :: Experiment : Electrostatic Force

You can easily test that like charges repel and unlike charges attract each other by doing a very simple experiment.

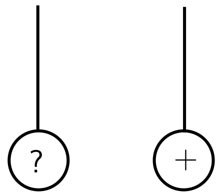
Take a glass rod and rub it with a piece of silk, then hang it from its middle with a piece string so that it is free to move. If you then bring another glass rod which you have also charged in the same way next to it, you will see the rod on the string turn *away* from the rod in your hand i.e. it is **repelled**. If, however, you take a plastic rod, rub it with a piece of fur and then bring it close to the rod on the string, you will see the rod on the string turn *towards* the rod in your hand i.e. it is **attracted**.



This happens because when you rub the glass with silk, tiny amounts of negative charge are transferred from the glass onto the silk, which causes the glass to have less negative charge than positive charge, making it **positively charged**. When you rub the plastic rod with the fur, you transfer tiny amounts of negative charge onto the rod and so it has more negative charge than positive charge on it, making it **negatively charged**.

Worked Example 41: Application of electrostatic forces

Question: Two charged metal spheres hang from strings and are free to move as shown in the picture below. The right hand sphere is positively charged. The charge on the left hand sphere is unknown.



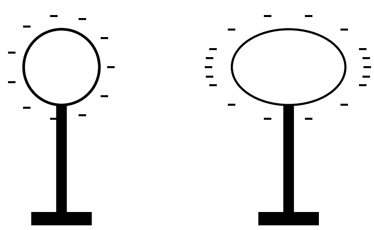
The left sphere is now brought close to the right sphere.

1. If the left hand sphere swings towards the right hand sphere, what can you say about the charge on the left sphere and why?
2. If the left hand sphere swings away from the right hand sphere, what can you say about the charge on the left sphere and why?

Answer
Step 1 : Identify what is known and what question you need to answer:
In the first case, we have a sphere with positive charge which is *attracting* the left charged sphere. We need to find the charge on the left sphere.
Step 2 : What concept is being used?
We are dealing with electrostatic forces between charged objects. Therefore, we know that *like* charges *repel* each other and *opposite* charges *attract* each other.
Step 3 : Use the concept to find the solution

1. In the first case, the positively charged sphere is attracting the left sphere. Since an electrostatic force between unlike charges is attractive, the left sphere must be *negatively* charged.
2. In the second case, the positively charged sphere repels the left sphere. Like charges repel each other. Therefore, the left sphere must now also be *positively* charged.

Extension: Electrostatic Force
The electrostatic force determines the arrangement of charge on the surface of conductors. When we place a charge on a spherical conductor the repulsive forces between the individual like charges cause them to spread uniformly over the surface of the sphere. However, for conductors with non-regular shapes, there is a concentration of charge near the point or points of the object.



This collection of charge can actually allow charge to leak off the conductor if the point is sharp enough. It is for this reason that buildings often have a lightning rod on the roof to remove any charge the building has collected. This minimises the possibility of the building being struck by lightning. This "spreading out" of charge would not occur if we were to place the charge on an insulator since charge cannot move in insulators.

Interesting Fact

The word 'electron' comes from the Greek word for amber. The ancient Greeks observed that if you rubbed a piece of amber, you could use it to pick up bits of straw.

9.6 Conductors and insulators

All atoms are electrically neutral i.e. they have the same amounts of negative and positive charge inside them. By convention, the electrons carry negative charge and the protons carry positive charge. The basic unit of charge, called the elementary charge, *e*, is the amount of charge carried by one electron.

All the matter and materials on earth are made up of atoms. Some materials allow electrons to move relatively freely through them (e.g. most metals, the human body). These materials are called **conductors**.

Other materials do not allow the charge carriers, the electrons, to move through them (e.g. plastic, glass). The electrons are bound to the atoms in the material. These materials are called **non-conductors** or **insulators**.

If an excess of charge is placed on an insulator, it will stay where it is put and there will be a concentration of charge in that area of the object. However, if an excess of charge is placed on a conductor, the like charges will repel each other and spread out over the surface of the object. When two conductors are made to touch, the total charge on them is shared between the two. If the two conductors are identical, then each conductor will be left with half of the total charge.

Extension: Charge and electrons

The basic unit of charge, namely the elementary charge is carried by the electron (equal to 1.602×10^{-19} C!). In a conducting material (e.g. copper), when the atoms bond to form the material, some of the outermost, loosely bound electrons become detached from the individual atoms and so become free to move around. The charge carried by these electrons can move around in the material. In insulators, there are very few, if any, free electrons and so the charge cannot move around in the material.

Worked Example 42: Conducting spheres and movement of charge

Question: I have 2 charged metal conducting spheres. Sphere A has a charge of -5 nC and sphere B has a charge of -3 nC. I then bring the spheres together so that they touch each other. Afterwards I move the two spheres apart so that they are no longer touching.

1. What happens to the charge on the two spheres?
2. What is the final charge on each sphere?

Answer

Step 1 : Identify what is known and what question/s we need to answer:

We have two identical negatively charged conducting spheres which are brought together to touch each other and then taken apart again. We need to explain what happens to the charge on each sphere and what the final charge on each sphere is after they are moved apart.

Step 2 : What concept is being used?

We know that the charge carriers in conductors are free to move around and that charge on a conductor spreads itself out on the surface of the conductor.

Step 3 : Use the concept to find the answer

1. When the two conducting spheres are brought together to touch, it is as though they become one single big conductor and the total charge of the two spheres spreads out across the whole surface of the touching spheres. When the spheres are moved apart again, each one is left with half of the total original charge.

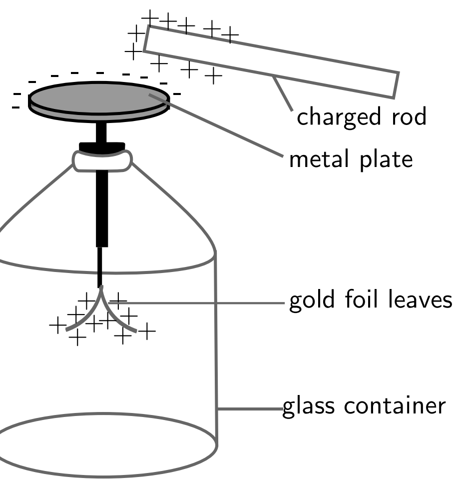
2. Before the spheres touch, the total charge is: -5 nC + (-3) nC = -8 nC. When they touch they share out the -8 nC across their whole surface. When they are removed from each other, each is left with half of the original charge:

$$-8 \text{ nC} /2 \quad = \quad -4 \text{ nC}$$

on each sphere.

9.6.1 The electroscope

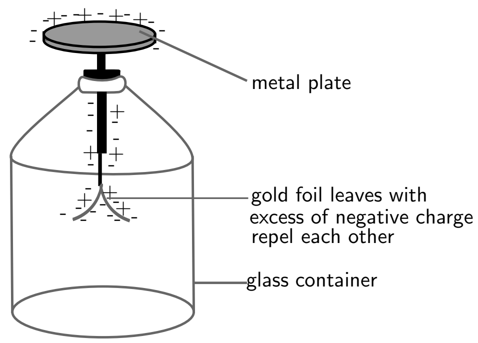
The electroscope is a very sensitive instrument which can be used to detect electric charge. A diagram of a gold leaf electroscope is shown the figure below. The electroscope consists of a glass container with a metal rod inside which has 2 thin pieces of gold foil attached. The other end of the metal rod has a metal plate attached to it outside the glass container.



The electroscope detects charge in the following way: A charged object, like the positively charged rod in the picture, is brought close to (but not touching) the neutral metal plate of the electroscope. This causes negative charge in the gold foil, metal rod, and metal plate, to be attracted to the positive rod. Because the metal (gold is a metal too!) is a conductor, the charge can move freely from the foil up the metal rod and onto the metal plate. There is now more negative charge on the plate and more positive charge on the gold foil leaves. This is called *inducing* a charge on the metal plate. It is important to remember that the electroscope is still neutral (the total positive and negative charges are the same), the charges have just been induced to *move* to different parts of the instrument! The induced positive charge on the gold leaves forces them apart since like charges repel! This is how we can tell that the rod is charged. If the rod is now moved away from the metal plate, the charge in the electroscope will spread itself out evenly again and the leaves will fall down again because there will no longer be an induced charge on them.

Grounding

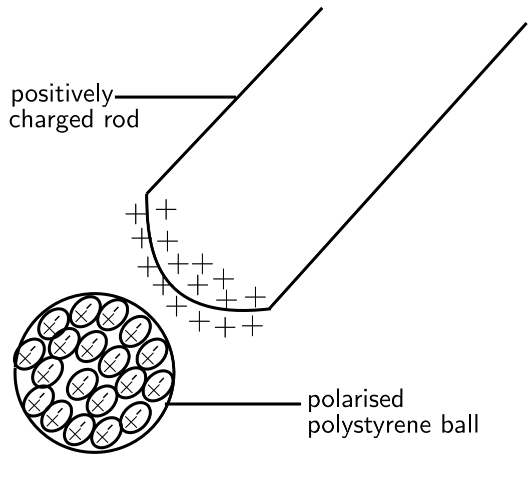
If you were to bring the charged rod close to the uncharged electroscope, and then you touched the metal plate with your finger at the same time, this would cause charge to flow up from the ground (the earth), through your body onto the metal plate. This is called **grounding**. The charge flowing onto the plate is opposite to the charge on the rod, since it is attracted to the rod. Therefore, for our picture, the charge flowing onto the plate would be negative. Now charge has been added to the electroscope. It is no longer neutral, but has an excess of negative charge. Now if we move the rod away, the leaves will remain apart because they have an excess of negative charge and they repel each other.



9.7 Attraction between charged and uncharged objects

9.7.1 Polarisation of Insulators

Unlike conductors, the electrons in insulators (non-conductors) are bound to the atoms of the insulator and cannot move around freely in the material. However, a charged object can still exert a force on a neutral insulator through the concept of **polarisation**.
If a positively charged rod is brought close to a neutral insulator such as polystyrene, it can attract the bound electrons to move round to the side of the atoms which is closest to the rod and cause the positive nuclei to move slightly to the opposite side of the atoms. This process is called *polarisation*. Although it is a very small (microscopic) effect, if there are many atoms and the polarised object is light (e.g. a small polystyrene ball), it can add up to enough force to be attracted onto the charged rod. Remember, that the polystyrene is *only* polarised, *not charged.* The polystyrene ball is still neutral since no charge was added or removed from it. The picture shows a not-to-scale view of the polarised atoms in the polystyrene ball:



Some materials are made up of molecules which are already polarised. These are molecules which have a more positive and a more negative side but are still neutral overall. Just as a polarised polystyrene ball can be attracted to a charged rod, these materials are also affected if brought close to a charged object.

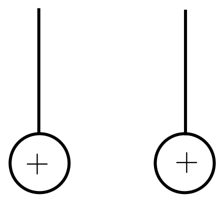
Water is an example of a substance which is made of polarised molecules. If a positively charged rod is brought close to a stream of water, the molecules can rotate so that the negative sides all line up towards the rod. The stream of water will then be attracted to the rod since opposite charges attract.

9.8 Summary

1. Objects can be **positively** charged, **negatively** charged or **neutral**.
2. Objects that are neutral have equal numbers of positive and negative charge.
3. Unlike charges are attracted to each other and like charges are repelled from each other.
4. Charge is neither created nor destroyed, it can only be transferred.
5. Charge is measured in coulombs (C).
6. Conductors allow charge to move through them easily.
7. Insulators do not allow charge to move through them easily.

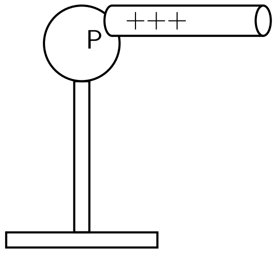
9.9 End of chapter exercise

1. What are the two types of charge called?
2. Provide evidence for the existence of two types of charge.
3. The electrostatic force between like charges is ????? while the electrostatic force between opposite charges is ?????.
4. I have two positively charged metal balls placed 2 m apart.
   4.1 Is the electrostatic force between the balls attractive or repulsive?
   4.2 If I now move the balls so that they are 1 m apart, what happens to the strength of the electrostatic force between them?
5. I have 2 charged spheres each hanging from string as shown in the picture below.

   Choose the correct answer from the options below: The spheres will
   5.1 swing towards each other due to the attractive electrostatic force between them.
   5.2 swing away from each other due to the attractive electrostatic force between them.
   5.3 swing towards each other due to the repulsive electrostatic force between them.
   5.4 swing away from each other due to the repulsive electrostatic force between them.
6. Describe how objects (insulators) can be charged by contact or rubbing.
7. You are given a perspex ruler and a piece of cloth.
   7.1 How would you charge the perspex ruler?
   7.2 Explain how the ruler becomes charged in terms of charge.
   7.3 How does the charged ruler attract small pieces of paper?

8. [IEB 2005/11 HG] An uncharged hollow metal sphere is placed on an insulating stand. A positively charged rod is brought up to touch the hollow metal sphere at P as shown in the diagram below. It is then moved away from the sphere.



Where is the excess charge distributed on the sphere after the rod has been removed?

8.1 It is still located at point P where the rod touched the sphere.

8.2 It is evenly distributed over the outer surface of the hollow sphere.

8.3 It is evenly distributed over the outer and inner surfaces of the hollow sphere.

8.4 No charge remains on the hollow sphere.

9. What is the process called where molecules in an uncharged object are caused to align in a particular direction due to an external charge?

10. Explain how an uncharged object can be attracted to a charged object. You should use diagrams to illustrate your answer.

11. Explain how a stream of water can be attracted to a charged rod.

Appendix A

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PREAMBLE

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