



FHSST Authors

**The Free High School Science Texts:
Textbooks for High School Students
Studying the Sciences
Physics
Grades 10 - 12**

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this a continuously evolving resource!

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Contents

I	Introduction	1
1	What is Physics?	3
II	Grade 10 - Physics	5
2	Units	9
2.1	Introduction	9
2.2	Unit Systems	9
2.2.1	SI Units	9
2.2.2	The Other Systems of Units	10
2.3	Writing Units as Words or Symbols	10
2.4	Combinations of SI Base Units	12
2.5	Rounding, Scientific Notation and Significant Figures	12
2.5.1	Rounding Off	12
2.5.2	Error Margins	13
2.5.3	Scientific Notation	13
2.5.4	Significant Figures	15
2.6	Prefixes of Base Units	15
2.7	The Importance of Units	17
2.8	How to Change Units	17
2.8.1	Two other useful conversions	19
2.9	A sanity test	19
2.10	Summary	19
2.11	End of Chapter Exercises	21
3	Motion in One Dimension - Grade 10	23
3.1	Introduction	23
3.2	Reference Point, Frame of Reference and Position	23
3.2.1	Frames of Reference	23
3.2.2	Position	25
3.3	Displacement and Distance	28
3.3.1	Interpreting Direction	29
3.3.2	Differences between Distance and Displacement	29
3.4	Speed, Average Velocity and Instantaneous Velocity	31

3.4.1	Differences between Speed and Velocity	35
3.5	Acceleration	38
3.6	Description of Motion	39
3.6.1	Stationary Object	40
3.6.2	Motion at Constant Velocity	41
3.6.3	Motion at Constant Acceleration	46
3.7	Summary of Graphs	48
3.8	Worked Examples	49
3.9	Equations of Motion	54
3.9.1	Finding the Equations of Motion	54
3.10	Applications in the Real-World	59
3.11	Summary	61
3.12	End of Chapter Exercises: Motion in One Dimension	62
4	Gravity and Mechanical Energy - Grade 10	67
4.1	Weight	67
4.1.1	Differences between Mass and Weight	68
4.2	Acceleration due to Gravity	69
4.2.1	Gravitational Fields	69
4.2.2	Free fall	69
4.3	Potential Energy	73
4.4	Kinetic Energy	75
4.4.1	Checking units	77
4.5	Mechanical Energy	78
4.5.1	Conservation of Mechanical Energy	78
4.5.2	Using the Law of Conservation of Energy	79
4.6	Energy graphs	82
4.7	Summary	83
4.8	End of Chapter Exercises: Gravity and Mechanical Energy	84
5	Transverse Pulses - Grade 10	87
5.1	Introduction	87
5.2	What is a <i>medium</i> ?	87
5.3	What is a <i>pulse</i> ?	87
5.3.1	Pulse Length and Amplitude	88
5.3.2	Pulse Speed	89
5.4	Graphs of Position and Velocity	90
5.4.1	Motion of a Particle of the Medium	90
5.4.2	Motion of the Pulse	92
5.5	Transmission and Reflection of a Pulse at a Boundary	96
5.6	Reflection of a Pulse from Fixed and Free Ends	97
5.6.1	Reflection of a Pulse from a Fixed End	97

5.6.2	Reflection of a Pulse from a Free End	98
5.7	Superposition of Pulses	99
5.8	Exercises - Transverse Pulses	102
6	Transverse Waves - Grade 10	105
6.1	Introduction	105
6.2	What is a <i>transverse wave</i> ?	105
6.2.1	Peaks and Troughs	106
6.2.2	Amplitude and Wavelength	107
6.2.3	Points in Phase	109
6.2.4	Period and Frequency	110
6.2.5	Speed of a Transverse Wave	111
6.3	Graphs of Particle Motion	115
6.4	Standing Waves and Boundary Conditions	118
6.4.1	Reflection of a Transverse Wave from a Fixed End	118
6.4.2	Reflection of a Transverse Wave from a Free End	118
6.4.3	Standing Waves	118
6.4.4	Nodes and anti-nodes	122
6.4.5	Wavelengths of Standing Waves with Fixed and Free Ends	122
6.4.6	Superposition and Interference	125
6.5	Summary	127
6.6	Exercises	127
7	Geometrical Optics - Grade 10	129
7.1	Introduction	129
7.2	Light Rays	129
7.2.1	Shadows	132
7.2.2	Ray Diagrams	132
7.3	Reflection	132
7.3.1	Terminology	133
7.3.2	Law of Reflection	133
7.3.3	Types of Reflection	135
7.4	Refraction	137
7.4.1	Refractive Index	139
7.4.2	Snell's Law	139
7.4.3	Apparent Depth	143
7.5	Mirrors	146
7.5.1	Image Formation	146
7.5.2	Plane Mirrors	147
7.5.3	Ray Diagrams	148
7.5.4	Spherical Mirrors	150
7.5.5	Concave Mirrors	150

7.5.6	Convex Mirrors	153
7.5.7	Summary of Properties of Mirrors	154
7.5.8	Magnification	154
7.6	Total Internal Reflection and Fibre Optics	156
7.6.1	Total Internal Reflection	156
7.6.2	Fibre Optics	161
7.7	Summary	163
7.8	Exercises	164
8	Magnetism - Grade 10	167
8.1	Introduction	167
8.2	Magnetic fields	167
8.3	Permanent magnets	169
8.3.1	The poles of permanent magnets	169
8.3.2	Magnetic attraction and repulsion	169
8.3.3	Representing magnetic fields	170
8.4	The compass and the earth's magnetic field	173
8.4.1	The earth's magnetic field	175
8.5	Summary	175
8.6	End of chapter exercises	176
9	Electrostatics - Grade 10	177
9.1	Introduction	177
9.2	Two kinds of charge	177
9.3	Unit of charge	177
9.4	Conservation of charge	177
9.5	Force between Charges	178
9.6	Conductors and insulators	181
9.6.1	The electroscope	182
9.7	Attraction between charged and uncharged objects	183
9.7.1	Polarisation of Insulators	183
9.8	Summary	184
9.9	End of chapter exercise	184
10	Electric Circuits - Grade 10	187
10.1	Electric Circuits	187
10.1.1	Closed circuits	187
10.1.2	Representing electric circuits	188
10.2	Potential Difference	192
10.2.1	Potential Difference	192
10.2.2	Potential Difference and Parallel Resistors	193
10.2.3	Potential Difference and Series Resistors	194
10.2.4	Ohm's Law	194

10.2.5 EMF	195
10.3 Current	198
10.3.1 Flow of Charge	198
10.3.2 Current	198
10.3.3 Series Circuits	199
10.3.4 Parallel Circuits	200
10.4 Resistance	202
10.4.1 What causes resistance?	202
10.4.2 Resistors in electric circuits	202
10.5 Instruments to Measure voltage, current and resistance	204
10.5.1 Voltmeter	204
10.5.2 Ammeter	204
10.5.3 Ohmmeter	204
10.5.4 Meters Impact on Circuit	205
10.6 Exercises - Electric circuits	205
III Grade 11 - Physics	209
11 Vectors	211
11.1 Introduction	211
11.2 Scalars and Vectors	211
11.3 Notation	211
11.3.1 Mathematical Representation	212
11.3.2 Graphical Representation	212
11.4 Directions	212
11.4.1 Relative Directions	212
11.4.2 Compass Directions	213
11.4.3 Bearing	213
11.5 Drawing Vectors	214
11.6 Mathematical Properties of Vectors	215
11.6.1 Adding Vectors	215
11.6.2 Subtracting Vectors	217
11.6.3 Scalar Multiplication	218
11.7 Techniques of Vector Addition	218
11.7.1 Graphical Techniques	218
11.7.2 Algebraic Addition and Subtraction of Vectors	223
11.8 Components of Vectors	228
11.8.1 Vector addition using components	231
11.8.2 Summary	235
11.8.3 End of chapter exercises: Vectors	236
11.8.4 End of chapter exercises: Vectors - Long questions	237

12 Force, Momentum and Impulse - Grade 11	239
12.1 Introduction	239
12.2 Force	239
12.2.1 What is a <i>force</i> ?	239
12.2.2 Examples of Forces in Physics	240
12.2.3 Systems and External Forces	241
12.2.4 Force Diagrams	242
12.2.5 Free Body Diagrams	243
12.2.6 Finding the Resultant Force	244
12.2.7 Exercise	246
12.3 Newton's Laws	246
12.3.1 Newton's First Law	247
12.3.2 Newton's Second Law of Motion	249
12.3.3 Exercise	261
12.3.4 Newton's Third Law of Motion	263
12.3.5 Exercise	267
12.3.6 Different types of forces	268
12.3.7 Exercise	275
12.3.8 Forces in equilibrium	276
12.3.9 Exercise	279
12.4 Forces between Masses	282
12.4.1 Newton's Law of Universal Gravitation	282
12.4.2 Comparative Problems	284
12.4.3 Exercise	286
12.5 Momentum and Impulse	287
12.5.1 Vector Nature of Momentum	290
12.5.2 Exercise	291
12.5.3 Change in Momentum	291
12.5.4 Exercise	293
12.5.5 Newton's Second Law revisited	293
12.5.6 Impulse	294
12.5.7 Exercise	296
12.5.8 Conservation of Momentum	297
12.5.9 Physics in Action: Impulse	300
12.5.10 Exercise	301
12.6 Torque and Levers	302
12.6.1 Torque	302
12.6.2 Mechanical Advantage and Levers	305
12.6.3 Classes of levers	307
12.6.4 Exercise	308
12.7 Summary	309
12.8 End of Chapter exercises	310

13 Geometrical Optics - Grade 11	327
13.1 Introduction	327
13.2 Lenses	327
13.2.1 Converging Lenses	329
13.2.2 Diverging Lenses	340
13.2.3 Summary of Image Properties	343
13.3 The Human Eye	344
13.3.1 Structure of the Eye	345
13.3.2 Defects of Vision	346
13.4 Gravitational Lenses	347
13.5 Telescopes	347
13.5.1 Refracting Telescopes	347
13.5.2 Reflecting Telescopes	348
13.5.3 Southern African Large Telescope	348
13.6 Microscopes	349
13.7 Summary	351
13.8 Exercises	352
14 Longitudinal Waves - Grade 11	355
14.1 Introduction	355
14.2 What is a <i>longitudinal wave</i> ?	355
14.3 Characteristics of Longitudinal Waves	356
14.3.1 Compression and Rarefaction	356
14.3.2 Wavelength and Amplitude	357
14.3.3 Period and Frequency	357
14.3.4 Speed of a Longitudinal Wave	358
14.4 Graphs of Particle Position, Displacement, Velocity and Acceleration	359
14.5 Sound Waves	360
14.6 Seismic Waves	361
14.7 Summary - Longitudinal Waves	361
14.8 Exercises - Longitudinal Waves	362
15 Sound - Grade 11	363
15.1 Introduction	363
15.2 Characteristics of a Sound Wave	363
15.2.1 Pitch	364
15.2.2 Loudness	364
15.2.3 Tone	364
15.3 Speed of Sound	365
15.4 Physics of the Ear and Hearing	365
15.4.1 Intensity of Sound	366
15.5 Ultrasound	367

15.6 SONAR	368
15.6.1 Echolocation	368
15.7 Summary	369
15.8 Exercises	369
16 The Physics of Music - Grade 11	373
16.1 Introduction	373
16.2 Standing Waves in String Instruments	373
16.3 Standing Waves in Wind Instruments	377
16.4 Resonance	382
16.5 Music and Sound Quality	384
16.6 Summary - The Physics of Music	385
16.7 End of Chapter Exercises	386
17 Electrostatics - Grade 11	387
17.1 Introduction	387
17.2 Forces between charges - Coulomb's Law	387
17.3 Electric field around charges	392
17.3.1 Electric field lines	393
17.3.2 Positive charge acting on a test charge	393
17.3.3 Combined charge distributions	394
17.3.4 Parallel plates	397
17.4 Electrical potential energy and potential	400
17.4.1 Electrical potential	400
17.4.2 Real-world application: lightning	402
17.5 Capacitance and the parallel plate capacitor	403
17.5.1 Capacitors and capacitance	403
17.5.2 Dielectrics	404
17.5.3 Physical properties of the capacitor and capacitance	404
17.5.4 Electric field in a capacitor	405
17.6 Capacitor as a circuit device	406
17.6.1 A capacitor in a circuit	406
17.6.2 Real-world applications: capacitors	407
17.7 Summary	407
17.8 Exercises - Electrostatics	407
18 Electromagnetism - Grade 11	413
18.1 Introduction	413
18.2 Magnetic field associated with a current	413
18.2.1 Real-world applications	418
18.3 Current induced by a changing magnetic field	420
18.3.1 Real-life applications	422
18.4 Transformers	423

18.4.1 Real-world applications	425
18.5 Motion of a charged particle in a magnetic field	425
18.5.1 Real-world applications	426
18.6 Summary	427
18.7 End of chapter exercises	427
19 Electric Circuits - Grade 11	429
19.1 Introduction	429
19.2 Ohm's Law	429
19.2.1 Definition of Ohm's Law	429
19.2.2 Ohmic and non-ohmic conductors	431
19.2.3 Using Ohm's Law	432
19.3 Resistance	433
19.3.1 Equivalent resistance	433
19.3.2 Use of Ohm's Law in series and parallel Circuits	438
19.3.3 Batteries and internal resistance	440
19.4 Series and parallel networks of resistors	442
19.5 Wheatstone bridge	445
19.6 Summary	447
19.7 End of chapter exercise	447
20 Electronic Properties of Matter - Grade 11	451
20.1 Introduction	451
20.2 Conduction	451
20.2.1 Metals	453
20.2.2 Insulator	453
20.2.3 Semi-conductors	454
20.3 Intrinsic Properties and Doping	454
20.3.1 Surplus	455
20.3.2 Deficiency	455
20.4 The p-n junction	457
20.4.1 Differences between p- and n-type semi-conductors	457
20.4.2 The p-n Junction	457
20.4.3 Unbiased	457
20.4.4 Forward biased	457
20.4.5 Reverse biased	458
20.4.6 Real-World Applications of Semiconductors	458
20.5 End of Chapter Exercises	459
IV Grade 12 - Physics	461
21 Motion in Two Dimensions - Grade 12	463
21.1 Introduction	463

21.2 Vertical Projectile Motion	463
21.2.1 Motion in a Gravitational Field	463
21.2.2 Equations of Motion	464
21.2.3 Graphs of Vertical Projectile Motion	467
21.3 Conservation of Momentum in Two Dimensions	475
21.4 Types of Collisions	480
21.4.1 Elastic Collisions	480
21.4.2 Inelastic Collisions	485
21.5 Frames of Reference	490
21.5.1 Introduction	490
21.5.2 What is a <i>frame of reference</i> ?	491
21.5.3 Why are frames of reference important?	491
21.5.4 Relative Velocity	491
21.6 Summary	494
21.7 End of chapter exercises	495
22 Mechanical Properties of Matter - Grade 12	503
22.1 Introduction	503
22.2 Deformation of materials	503
22.2.1 Hooke's Law	503
22.2.2 Deviation from Hooke's Law	506
22.3 Elasticity, plasticity, fracture, creep	508
22.3.1 Elasticity and plasticity	508
22.3.2 Fracture, creep and fatigue	508
22.4 Failure and strength of materials	509
22.4.1 The properties of matter	509
22.4.2 Structure and failure of materials	509
22.4.3 Controlling the properties of materials	509
22.4.4 Steps of Roman Swordsmithing	510
22.5 Summary	511
22.6 End of chapter exercise	511
23 Work, Energy and Power - Grade 12	513
23.1 Introduction	513
23.2 Work	513
23.3 Energy	519
23.3.1 External and Internal Forces	519
23.3.2 Capacity to do Work	520
23.4 Power	525
23.5 Important Equations and Quantities	529
23.6 End of Chapter Exercises	529

24 Doppler Effect - Grade 12	533
24.1 Introduction	533
24.2 The Doppler Effect with Sound and Ultrasound	533
24.2.1 Ultrasound and the Doppler Effect	537
24.3 The Doppler Effect with Light	537
24.3.1 The Expanding Universe	538
24.4 Summary	539
24.5 End of Chapter Exercises	539
25 Colour - Grade 12	541
25.1 Introduction	541
25.2 Colour and Light	541
25.2.1 Dispersion of white light	544
25.3 Addition and Subtraction of Light	544
25.3.1 Additive Primary Colours	544
25.3.2 Subtractive Primary Colours	545
25.3.3 Complementary Colours	546
25.3.4 Perception of Colour	546
25.3.5 Colours on a Television Screen	547
25.4 Pigments and Paints	548
25.4.1 Colour of opaque objects	548
25.4.2 Colour of transparent objects	548
25.4.3 Pigment primary colours	549
25.5 End of Chapter Exercises	550
26 2D and 3D Wavefronts - Grade 12	553
26.1 Introduction	553
26.2 Wavefronts	553
26.3 The Huygens Principle	554
26.4 Interference	556
26.5 Diffraction	557
26.5.1 Diffraction through a Slit	558
26.6 Shock Waves and Sonic Booms	562
26.6.1 Subsonic Flight	563
26.6.2 Supersonic Flight	563
26.6.3 Mach Cone	566
26.7 End of Chapter Exercises	568
27 Wave Nature of Matter - Grade 12	571
27.1 Introduction	571
27.2 de Broglie Wavelength	571
27.3 The Electron Microscope	574
27.3.1 Disadvantages of an Electron Microscope	577

27.3.2	Uses of Electron Microscopes	577
27.4	End of Chapter Exercises	578
28	Electrodynamics - Grade 12	579
28.1	Introduction	579
28.2	Electrical machines - generators and motors	579
28.2.1	Electrical generators	580
28.2.2	Electric motors	582
28.2.3	Real-life applications	582
28.2.4	Exercise - generators and motors	584
28.3	Alternating Current	585
28.3.1	Exercise - alternating current	586
28.4	Capacitance and inductance	586
28.4.1	Capacitance	586
28.4.2	Inductance	586
28.4.3	Exercise - capacitance and inductance	588
28.5	Summary	588
28.6	End of chapter exercise	589
29	Electronics - Grade 12	591
29.1	Introduction	591
29.2	Capacitive and Inductive Circuits	591
29.3	Filters and Signal Tuning	596
29.3.1	Capacitors and Inductors as Filters	596
29.3.2	LRC Circuits, Resonance and Signal Tuning	596
29.4	Active Circuit Elements	599
29.4.1	The Diode	599
29.4.2	The Light Emitting Diode (LED)	601
29.4.3	Transistor	603
29.4.4	The Operational Amplifier	607
29.5	The Principles of Digital Electronics	609
29.5.1	Logic Gates	610
29.6	Using and Storing Binary Numbers	616
29.6.1	Binary numbers	616
29.6.2	Counting circuits	617
29.6.3	Storing binary numbers	619
30	EM Radiation	625
30.1	Introduction	625
30.2	Particle/wave nature of electromagnetic radiation	625
30.3	The wave nature of electromagnetic radiation	626
30.4	Electromagnetic spectrum	626
30.5	The particle nature of electromagnetic radiation	629

30.5.1 Exercise - particle nature of EM waves	630
30.6 Penetrating ability of electromagnetic radiation	631
30.6.1 Ultraviolet(UV) radiation and the skin	631
30.6.2 Ultraviolet radiation and the eyes	632
30.6.3 X-rays	632
30.6.4 Gamma-rays	632
30.6.5 Exercise - Penetrating ability of EM radiation	633
30.7 Summary	633
30.8 End of chapter exercise	633
31 Optical Phenomena and Properties of Matter - Grade 12	635
31.1 Introduction	635
31.2 The transmission and scattering of light	635
31.2.1 Energy levels of an electron	635
31.2.2 Interaction of light with metals	636
31.2.3 Why is the sky blue?	637
31.3 The photoelectric effect	638
31.3.1 Applications of the photoelectric effect	640
31.3.2 Real-life applications	642
31.4 Emission and absorption spectra	643
31.4.1 Emission Spectra	643
31.4.2 Absorption spectra	644
31.4.3 Colours and energies of electromagnetic radiation	646
31.4.4 Applications of emission and absorption spectra	648
31.5 Lasers	650
31.5.1 How a laser works	652
31.5.2 A simple laser	654
31.5.3 Laser applications and safety	655
31.6 Summary	656
31.7 End of chapter exercise	657
V Exercises	659
32 Exercises	661
VI Essays	663
Essay 1: Energy and electricity. Why the fuss?	665
33 Essay: How a cell phone works	671
34 Essay: How a Physiotherapist uses the Concept of Levers	673
35 Essay: How a Pilot Uses Vectors	675

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677

Chapter 9

Electrostatics - Grade 10

9.1 Introduction

Electrostatics is the study of electric charge which is static (not moving).

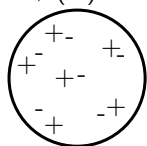
9.2 Two kinds of charge

All objects surrounding us (including people!) contain large amounts of electric charge. There are two types of electric charge: **positive** charge and **negative** charge. If the same amounts of negative and positive charge are brought together, they neutralise each other and there is no net charge. **Neutral** objects are objects which contain positive and negative charges, but in equal numbers. However, if there is a little bit more of one type of charge than the other on the object then the object is said to be **electrically charged**. The picture below shows what the distribution of charges might look like for a neutral, positively charged and negatively charged object.

There are:

6 positive charges and
6 negative charges

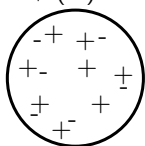
$$6 + (-6) = 0$$



There is zero net charge:
The object is neutral

8 positive charges and
6 negative charges

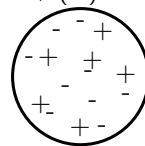
$$8 + (-6) = 2$$



The net charge is +2
The object is positively charged

6 positive charges and
9 negative charges

$$6 + (-9) = -3$$



The net charge is -3

The object is negatively charged

9.3 Unit of charge

Charge is measured in units called **coulombs (C)**. A coulomb of charge is a very large charge. In electrostatics we therefore often work with charge in microcoulombs ($1 \mu\text{C} = 1 \times 10^{-6} \text{ C}$) and nanocoulombs ($1 \text{ nC} = 1 \times 10^{-9} \text{ C}$).

9.4 Conservation of charge

Objects can become charged by contact or by rubbing them. This means that they can gain extra negative or positive charge. Charging happens when you, for example, rub your feet against the carpet. When you then touch something metallic or another person, you will feel a shock as

the excess charge that you have collected is *discharged*.



Important: Charge, just like energy, cannot be created or destroyed. We say that charge is **conserved**.

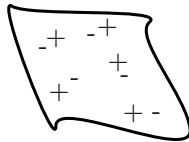
When you rub your feet against the carpet, negative charge is transferred to you from the carpet. The carpet will then become positively charged by the *same amount*.

Another example is to take two *neutral* objects such as a plastic ruler and a cotton cloth (handkerchief). To begin, the two objects are neutral (i.e. have the same amounts of positive and negative charge.)

BEFORE rubbing:



The ruler has 9 positive charges and 9 negative charges



The neutral cotton cloth has 5 positive charges and 5 negative charges

The total number of charges is:

$$(9+5)=14 \text{ positive charges}$$

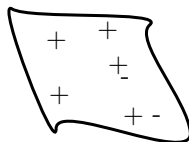
$$(9+5)=14 \text{ negative charges}$$

Now, if the cotton cloth is used to rub the ruler, negative charge is transferred *from* the cloth *to* the ruler. The ruler is now *negatively* charged and the cloth is *positively* charged. If you count up all the positive and negative charges at the beginning and the end, there are still the same amount. i.e. total charge has been *conserved*!

AFTER rubbing:



The ruler has 9 positive charges and 12 negative charges
It is now negatively charged.



The cotton cloth has 5 positive charges and 2 negative charges.
It is now positively charged.

The total number of charges is:

$$(9+5)=14 \text{ positive charges}$$

$$(12+2)=14 \text{ negative charges}$$

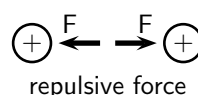
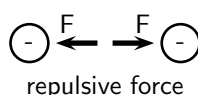
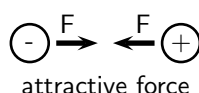
Charges have been transferred from the cloth to the ruler BUT total charge has been conserved!

9.5 Force between Charges

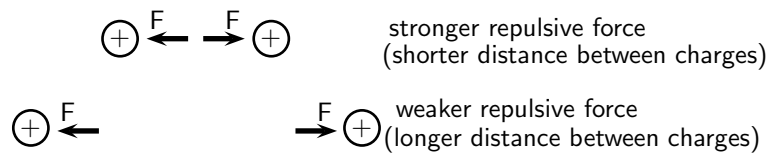
The force exerted by non-moving (static) charges on each other is called the **electrostatic force**. The electrostatic force between:

- **like** charges is **repulsive**
- **opposite** (unlike) charges is **attractive**.

In other words, like charges repel each other while opposite charges attract each other. This is different to the gravitational force which is only attractive.



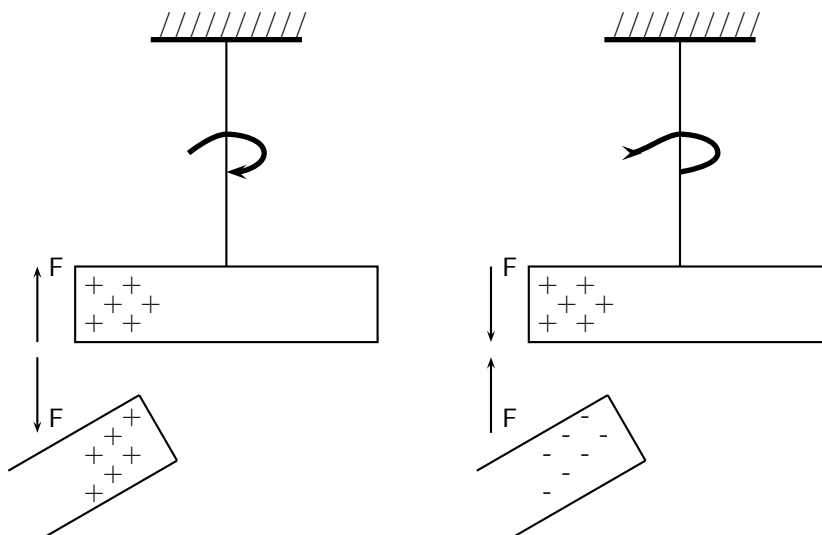
The *closer* together the charges are, the *stronger* the electrostatic force between them.



Activity :: Experiment : Electrostatic Force

You can easily test that like charges repel and unlike charges attract each other by doing a very simple experiment.

Take a glass rod and rub it with a piece of silk, then hang it from its middle with a piece of string so that it is free to move. If you then bring another glass rod which you have also charged in the same way next to it, you will see the rod on the string turn *away* from the rod in your hand i.e. it is **repelled**. If, however, you take a plastic rod, rub it with a piece of fur and then bring it close to the rod on the string, you will see the rod on the string turn *towards* the rod in your hand i.e. it is **attracted**.

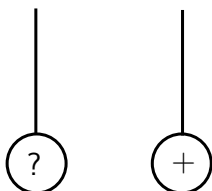


This happens because when you rub the glass with silk, tiny amounts of negative charge are transferred from the glass onto the silk, which causes the glass to have less negative charge than positive charge, making it **positively charged**. When you rub the plastic rod with the fur, you transfer tiny amounts of negative charge onto the rod and so it has more negative charge than positive charge on it, making it **negatively charged**.



Worked Example 41: Application of electrostatic forces

Question: Two charged metal spheres hang from strings and are free to move as shown in the picture below. The right hand sphere is positively charged. The charge on the left hand sphere is unknown.



The left sphere is now brought close to the right sphere.

1. If the left hand sphere swings towards the right hand sphere, what can you say about the charge on the left sphere and why?
2. If the left hand sphere swings away from the right hand sphere, what can you say about the charge on the left sphere and why?

Answer

Step 1 : Identify what is known and what question you need to answer:

In the first case, we have a sphere with positive charge which is *attracting* the left charged sphere. We need to find the charge on the left sphere.

Step 2 : What concept is being used?

We are dealing with electrostatic forces between charged objects. Therefore, we know that *like charges repel* each other and *opposite charges attract* each other.

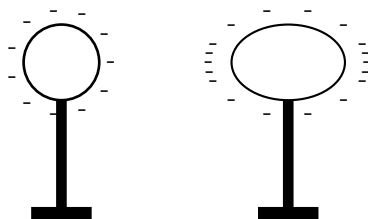
Step 3 : Use the concept to find the solution

1. In the first case, the positively charged sphere is attracting the left sphere. Since an electrostatic force between unlike charges is attractive, the left sphere must be *negatively* charged.
2. In the second case, the positively charged sphere repels the left sphere. Like charges repel each other. Therefore, the left sphere must now also be *positively* charged.

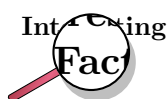


Extension: Electrostatic Force

The electrostatic force determines the arrangement of charge on the surface of conductors. When we place a charge on a spherical conductor the repulsive forces between the individual like charges cause them to spread uniformly over the surface of the sphere. However, for conductors with non-regular shapes, there is a concentration of charge near the point or points of the object.



This collection of charge can actually allow charge to leak off the conductor if the point is sharp enough. It is for this reason that buildings often have a lightning rod on the roof to remove any charge the building has collected. This minimises the possibility of the building being struck by lightning. This “spreading out” of charge would not occur if we were to place the charge on an insulator since charge cannot move in insulators.



The word 'electron' comes from the Greek word for amber. The ancient Greeks observed that if you rubbed a piece of amber, you could use it to pick up bits of straw.

9.6 Conductors and insulators

All atoms are electrically neutral i.e. they have the same amounts of negative and positive charge inside them. By convention, the electrons carry negative charge and the protons carry positive charge. The basic unit of charge, called the elementary charge, e , is the amount of charge carried by one electron.

All the matter and materials on earth are made up of atoms. Some materials allow electrons to move relatively freely through them (e.g. most metals, the human body). These materials are called **conductors**.

Other materials do not allow the charge carriers, the electrons, to move through them (e.g. plastic, glass). The electrons are bound to the atoms in the material. These materials are called **non-conductors** or **insulators**.

If an excess of charge is placed on an insulator, it will stay where it is put and there will be a concentration of charge in that area of the object. However, if an excess of charge is placed on a conductor, the like charges will repel each other and spread out over the surface of the object. When two conductors are made to touch, the total charge on them is shared between the two. If the two conductors are identical, then each conductor will be left with half of the total charge.



Extension: Charge and electrons

The basic unit of charge, namely the elementary charge is carried by the electron (equal to 1.602×10^{-19} C!). In a conducting material (e.g. copper), when the atoms bond to form the material, some of the outermost, loosely bound electrons become detached from the individual atoms and so become free to move around. The charge carried by these electrons can move around in the material. In insulators, there are very few, if any, free electrons and so the charge cannot move around in the material.



Worked Example 42: Conducting spheres and movement of charge

Question: I have 2 charged metal conducting spheres. Sphere A has a charge of -5 nC and sphere B has a charge of -3 nC. I then bring the spheres together so that they touch each other. Afterwards I move the two spheres apart so that they are no longer touching.

1. What happens to the charge on the two spheres?
2. What is the final charge on each sphere?

Answer

Step 1 : Identify what is known and what question/s we need to answer:

We have two identical negatively charged conducting spheres which are brought together to touch each other and then taken apart again. We need to explain what

happens to the charge on each sphere and what the final charge on each sphere is after they are moved apart.

Step 2 : What concept is being used?

We know that the charge carriers in conductors are free to move around and that charge on a conductor spreads itself out on the surface of the conductor.

Step 3 : Use the concept to find the answer

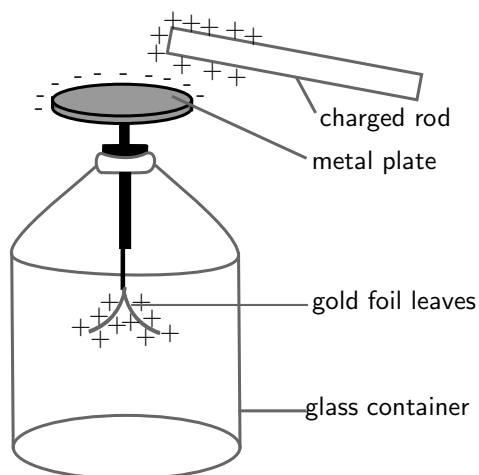
1. When the two conducting spheres are brought together to touch, it is as though they become one single big conductor and the total charge of the two spheres spreads out across the whole surface of the touching spheres. When the spheres are moved apart again, each one is left with half of the total original charge.
2. Before the spheres touch, the total charge is: $-5 \text{ nC} + (-3) \text{ nC} = -8 \text{ nC}$. When they touch they share out the -8 nC across their whole surface. When they are removed from each other, each is left with half of the original charge:

$$-8 \text{ nC} / 2 = -4 \text{ nC}$$

on each sphere.

9.6.1 The electroscope

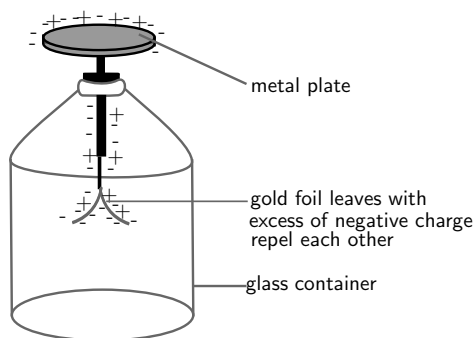
The electroscope is a very sensitive instrument which can be used to detect electric charge. A diagram of a gold leaf electroscope is shown in the figure below. The electroscope consists of a glass container with a metal rod inside which has 2 thin pieces of gold foil attached. The other end of the metal rod has a metal plate attached to it outside the glass container.



The electroscope detects charge in the following way: A charged object, like the positively charged rod in the picture, is brought close to (but not touching) the neutral metal plate of the electroscope. This causes negative charge in the gold foil, metal rod, and metal plate, to be attracted to the positive rod. Because the metal (gold is a metal too!) is a conductor, the charge can move freely from the foil up the metal rod and onto the metal plate. There is now more negative charge on the plate and more positive charge on the gold foil leaves. This is called *inducing* a charge on the metal plate. It is important to remember that the electroscope is still neutral (the total positive and negative charges are the same), the charges have just been induced to *move* to different parts of the instrument! The induced positive charge on the gold leaves forces them apart since like charges repel! This is how we can tell that the rod is charged. If the rod is now moved away from the metal plate, the charge in the electroscope will spread itself out evenly again and the leaves will fall down again because there will no longer be an induced charge on them.

Grounding

If you were to bring the charged rod close to the uncharged electroscope, and then you touched the metal plate with your finger at the same time, this would cause charge to flow up from the ground (the earth), through your body onto the metal plate. This is called **grounding**. The charge flowing onto the plate is opposite to the charge on the rod, since it is attracted to the rod. Therefore, for our picture, the charge flowing onto the plate would be negative. Now charge has been added to the electroscope. It is no longer neutral, but has an excess of negative charge. Now if we move the rod away, the leaves will remain apart because they have an excess of negative charge and they repel each other.

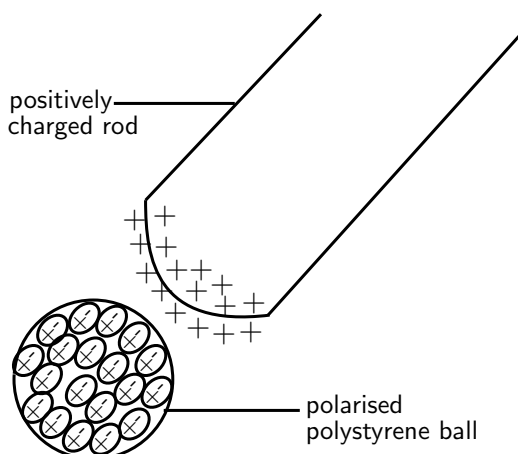


9.7 Attraction between charged and uncharged objects

9.7.1 Polarisation of Insulators

Unlike conductors, the electrons in insulators (non-conductors) are bound to the atoms of the insulator and cannot move around freely in the material. However, a charged object can still exert a force on a neutral insulator through the concept of **polarisation**.

If a positively charged rod is brought close to a neutral insulator such as polystyrene, it can attract the bound electrons to move round to the side of the atoms which is closest to the rod and cause the positive nuclei to move slightly to the opposite side of the atoms. This process is called *polarisation*. Although it is a very small (microscopic) effect, if there are many atoms and the polarised object is light (e.g. a small polystyrene ball), it can add up to enough force to be attracted onto the charged rod. Remember, that the polystyrene is *only* polarised, *not* charged. The polystyrene ball is still neutral since no charge was added or removed from it. The picture shows a not-to-scale view of the polarised atoms in the polystyrene ball:



Some materials are made up of molecules which are already polarised. These are molecules which have a more positive and a more negative side but are still neutral overall. Just as a polarised polystyrene ball can be attracted to a charged rod, these materials are also affected if brought close to a charged object.

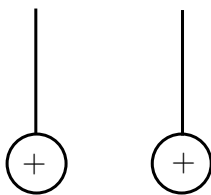
Water is an example of a substance which is made of polarised molecules. If a positively charged rod is brought close to a stream of water, the molecules can rotate so that the negative sides all line up towards the rod. The stream of water will then be attracted to the rod since opposite charges attract.

9.8 Summary

1. Objects can be **positively** charged, **negatively** charged or **neutral**.
2. Objects that are neutral have equal numbers of positive and negative charge.
3. Unlike charges are attracted to each other and like charges are repelled from each other.
4. Charge is neither created nor destroyed, it can only be transferred.
5. Charge is measured in coulombs (C).
6. Conductors allow charge to move through them easily.
7. Insulators do not allow charge to move through them easily.

9.9 End of chapter exercise

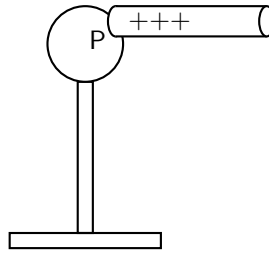
1. What are the two types of charge called?
2. Provide evidence for the existence of two types of charge.
3. The electrostatic force between like charges is ????? while the electrostatic force between opposite charges is ?????.
4. I have two positively charged metal balls placed 2 m apart.
 - 4.1 Is the electrostatic force between the balls attractive or repulsive?
 - 4.2 If I now move the balls so that they are 1 m apart, what happens to the strength of the electrostatic force between them?
5. I have 2 charged spheres each hanging from string as shown in the picture below.



Choose the correct answer from the options below: The spheres will

- 5.1 swing towards each other due to the attractive electrostatic force between them.
 - 5.2 swing away from each other due to the attractive electrostatic force between them.
 - 5.3 swing towards each other due to the repulsive electrostatic force between them.
 - 5.4 swing away from each other due to the repulsive electrostatic force between them.
6. Describe how objects (insulators) can be charged by contact or rubbing.
 7. You are given a perspex ruler and a piece of cloth.
 - 7.1 How would you charge the perspex ruler?
 - 7.2 Explain how the ruler becomes charged in terms of charge.
 - 7.3 How does the charged ruler attract small pieces of paper?

8. [IEB 2005/11 HG] An uncharged hollow metal sphere is placed on an insulating stand. A positively charged rod is brought up to touch the hollow metal sphere at P as shown in the diagram below. It is then moved away from the sphere.



- Where is the excess charge distributed on the sphere after the rod has been removed?
- 8.1 It is still located at point P where the rod touched the sphere.
 - 8.2 It is evenly distributed over the outer surface of the hollow sphere.
 - 8.3 It is evenly distributed over the outer and inner surfaces of the hollow sphere.
 - 8.4 No charge remains on the hollow sphere.
9. What is the process called where molecules in an uncharged object are caused to align in a particular direction due to an external charge?
10. Explain how an uncharged object can be attracted to a charged object. You should use diagrams to illustrate your answer.
11. Explain how a stream of water can be attracted to a charged rod.

Appendix A

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